Reliability of Porous Materials: Two Stochastic Approaches

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Abstract: The first decay shown by masonry subjected to an aggressive environment is the loss of surface material. The great randomness of this kind of decay phenomenon suggests studying it from a probabilistic point of view. Here, the evolution of decay in stone masonry has been investigated by applying two different approaches on experimental data collected on on-site models. Since the results are very similar, the choice of approach depends only on the available data and the monitoring time.

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Introduction

Structures subjected to an aggressive environment may suffer degradation of their component materials during their service life. The deterioration of material due to environmental attacks also affects structure, where the main aggressors are natural frost–defrost cycles. In the presence of moisture and/or capillary rise these cycles may produce the crystallization of salts inside the component materials (bricks, stones, and mortar), with the resulting loss of surface material. In historic buildings and monuments surface delamination is *already* an important damage (Binda et al. 1999). Experimental evidence has established that delamination compromises the mechanical characteristics of porous materials only on the wet–dry interface under the surface; in this case the loss of material is itself considered a damage. However, we must consider that when the delamination reaches a predetermined threshold $\overline{l}$, the reliability of the structural system is seriously compromised. But, when will this threshold be reached? And again: how to quantify the damage? The problem of quantifying surface damage without destructive testing and observation of its evolution in time without excessively long monitoring tests is an open question. The objective of our research has been to develop a procedure that can predict, in probabilistic terms, the evolution of surface decay in time, without a large set of experimental data.

The decay of materials is often investigated through experimental aging tests carried out in the laboratory. But the deterioration of materials in the environment is a process so complex that it cannot be completely reproduced in the laboratory. Reducing this complexity through appropriate modeling of the process seems to be a good approach (Binda and Molina 1990, Cranmer and Rickerson 1998, Bekker 1999). Binda and Baronio have produced models that evaluate the effects of salt crystallization on porous materials due to capillary rise. Their research, which is still in progress, is carried out in both indoor and outdoor laboratories (Binda et al. 1999). The experimental data recorded by this research group over a number of years form the basis of our investigation of reliability of masonry materials in time.

The great randomness involved in the deterioration of masonry suggests that this process be studied from a probabilistic point of view. Consequently, we have investigated the phenomenon from two different approaches:

1. The deterioration process $L(t)$ over time is defined as a stochastic process of the random variable (rv) l, where l is the loss of surface material.
2. The deterioration process $L(t)$ over time is defined as a stochastic process of the rv τ, where τ is the "lifetime" of the system. Since the deterioration process seems to renew itself at every loss of surface layer, it can be modeled as a renewal process.

Approaches 1 and 2 are discussed in detail in the second and third sections of this paper, respectively.

The two approaches have been applied to sets of experimental data collected on real stone–masonry walls subjected to salt decay. Our physical knowledge of the masonry deterioration process has suggested the use of Weibull distribution in both cases. The problems connected with this choice are also discussed, the two approaches are then compared in the fourth section.

As, the prediction of the time taken to reach a given level of damage is an important issue in planning strategies for the maintenance and repair of existing buildings, the paper provides details about this issue, as well.

Approach 1: Deterioration Process as Stochastic Process in Random Variable *l*

The deterioration process of stone masonry can be described through the parameter l, defined as the loss of material reached by the system at the time t^*. At every time t^*, the high randomness connected with the occurrence of the material decay in the natural environment leads us to consider l as a rv with a certain distribution of values (Fig. 1). Seen in this manner, the deterioration process can be interpreted as a stochastic process of the rv l. But the surface decay also depends on the instant t^* in which the

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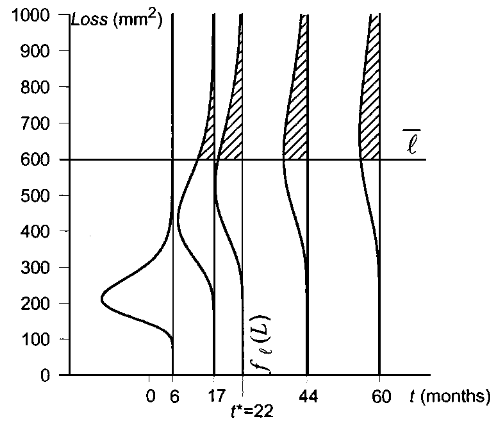

Fig. 1. Exceedance probability crossing threshold $\overline{l}$

deterioration is recorded. Therefore, at each time t^* the loss l (measured in millimeters squared) can be modeled with a probability density function (PDF) $f_l(L)$ which is dependent on the time $t^*=$constant value (e.g., $t_1=6$ months, $t_2=18$ months, and so on) and the rv l. In order to model $f_l(L)$, at a family of theoretical distributions must be chosen in every time t^*. Obviously, the distribution modeling a given phenomenon must be chosen on the basis of the physical aspects of the phenomenon itself, and of the characteristics of the distribution function in its "tail" where often no experimental data can be collected. The latter aspect of the matter can be investigated by analyzing the behavior of the failure rate function $\phi_l(L)$ at every time t^* connected with the chosen distribution function

$$\phi_l(L)dL = \Pr\{L < l \leqslant L + dL | l > L\} \quad \forall\, t^* \tag{1}$$

More details on this subject are given in Binda and Molina et al. (1990), Molina et al. (1996), and Garavaglia et al. (2002). The recorded experimental data considered here show a large dispersion around the average value of l. This is probably due to the randomness connected with the decay mechanism in a real environment. However, the loss seems to be contained within a certain range of values. It seems therefore correct to assume that, at a given time t^*, the probability of a loss ($L<l\leqslant L+dL$) decreases as the value L (magnitude of the loss) increases. This hypothesis, assumed as a satisfied (but not unique) physical interpretation of the decay process, leads us to model the loss l at the time t^* with a Lognormal PDF as follows:

$$f_l(L) = \frac{1}{L\sqrt{(2\pi\sigma)}} \exp\left\{-\frac{[\log(\eta L)]^2}{2\sigma}\right\} \quad \forall\, t^* \tag{2}$$

This family of distributions presents an immediate occurrence rate function Eq. (1) that decreases as the value of L increases; this fact seems to respect the physical interpretation of the decay process previously proposed.

Deterioration Process as Reliability Problem

The problem is to evaluate the probability that the system will reach, or exceed a given damage threshold $\overline{l}$ over time. Considering a significant damage $\overline{l}$ and the variable time needed to exceed it, the deterioration process can be treated as a reliability problem (Garavaglia et al. 2002).

Reliability $R(t)$ is concerned with the performance of a system over time, and is defined as the probability that the system does not fail by time t (e.g., Evans 1992). This definition is extended here, denoting by $\overline{R}(t)$ the probability that a system will exceed a given significant damage threshold $\overline{l}$ by time t. The random variable that is used to quantify reliability is $\overline{T}$, which is simply the time it takes to exceed damage $\overline{l}$. Thus, from this point of view, the reliability function is given by

$$\overline{R}(t) = \Pr(\overline{T} > t) = 1 - F_{\overline{T}}(t) \tag{3}$$

where $F_{\overline{T}}(t)=$distribution function for $\overline{T}$. Assuming that the density function $f_{\overline{T}}(t)$ exists for the rv $\overline{T}$, the failure rate function $\phi_{\overline{T}}(t)$ is given by (Evans 1992)

$$\phi_{\overline{T}}(t) = \frac{f_{\overline{T}}(t)}{\overline{R}(t)} \tag{4}$$

Computing $F_{\overline{T}}(t)$ for different damage levels $\overline{l}$ allows us to build the fragility curve for each $\overline{l}$. A fragility curve describes the probability of reaching or exceeding a given damage $\overline{l}$ over time (Singhal and Kerimidjian 1996). For a particular damage level $\overline{l}$ at a given time t^*, the probability that it will be reached can be seen as the area below the threshold $\overline{l}$ and the exceeding probability can be seen as the area above the threshold $\overline{l}$ (Fig. 1) (Garavaglia et al. 2002). This area can be calculated by using the survive function

$$\mathcal{F}_l(L) = 1 - F_l(L) \quad \forall\, t^* \tag{5}$$

where $F_l(L)=$cumulative distribution function of the PDF Eq. (2) at every t^*. The computation of $\mathcal{F}_l(L)$ at every t^* is possible with the use of any kind of computer code for numerical integration. The areas computed over different thresholds $\overline{l}$ provide the experimental fragility curves . Therefore, the evaluation for different t^* of the exceedance probability for each damage level $\overline{l}$, results in an experimental fragility curve for each chosen $\overline{l}$.

To model the experimental fragility curves, a Weibull distribution has been chosen (Molina et al. 1996; Cranmer and Richerson 1998; Bekker 1999; Garavaglia et al. 2002)

$$F_{\overline{T}}(t) = 1 - \exp[-(\rho t)^{\alpha}] \tag{6}$$

This distribution seems to provide a good interpretation of the physical phenomenon. In fact experimental evidence indicates that the failure rate function $\phi_{\overline{T}}(t)$ has a polynomial form and, consequently, must be (Garavaglia et al. 2002)

$$\phi_{\overline{T}}(t) = b_1 t^{b_2} \tag{7}$$

where $b_1>0$ and $-1<b_2<+\infty=$constants. Since

$$\overline{R}(t) = \exp\left\{-\int_0^t \phi_{\overline{T}}(u)du\right\} \tag{8}$$

and

$$f_{\overline{T}}(t) = \phi_{\overline{T}}(t)\exp\left\{-\int_0^t \phi_{\overline{T}}(u)du\right\} \tag{9}$$

and assuming: $b_1=\alpha\rho^{\alpha}$ and $b_2+1=\alpha$ (where α and $\rho=$constants) we obtain

$$\int_0^t \phi_{\overline{T}}(u)du = \frac{b_1 t^{b_2+1}}{b_2+1} = (\rho t)^\alpha \tag{10}$$

Therefore, by Eqs. (8) and (9)

$$\overline{R}(t) = \exp[-(\rho t)^\alpha] \tag{11}$$

and

$$f_{\overline{T}}(t) = \alpha\rho(\rho t)^{\alpha-1}\exp[-(\rho t)^\alpha] \tag{12}$$

which is definitely the well known PDF of Weibull distribution Eq. (6) (Cox 1962; Evans 1992).

Approach 2: Deterioration Process as Renewal Process in Random Variable τ

The deterioration process can also be seen as a loss of performance of the material (and the structural system too). Every time deterioration reaches a given level, the material suffers a "failure" and a loss of performance (Sarja 1996). This phenomenon can be described as the transition of the *system* (the material) through different service states characterized by different levels of performance. Thus the deterioration process (failure process) may be defined as a transition process (Binda and Molina 1990).

Every transition depends on (Sarja 1996; Bekker 1999):

- the magnitude of the attack (stress cycle); and
- the system's capacity to withstand this attack.

Both these parameters depend on a large number of time dependent random variable (rv); therefore it is correct to interpret the transition process as a stochastic process. Binda and Molina (1990) have shown that a possible rv, which can be assumed to describe a transition process, is the service lifetime τ_i defined as: "the waiting time spent by the material in the performance state i."

From this point of view the reliability function $R(t)$ can be defined as

$$R(t,t_0) = \Pr\{\tau_i > t\} = 1 - F_{\tau_i}(t,t_0) \tag{13}$$

where t_0 represents the age of the material when it enters state i, and $F_{\tau_i}(t)$=cumulative distribution function of the rv τ_i (Binda and Molina 1990).

When $F_{\tau_i}(t)$ is known, a stochastic dynamic process can be assumed to represent the process of material failure. Molina et al. (1996) have shown that the semi-Markov Processes (s-MP) (Howard 1971) seem to be suitable for describing the process of material failure. They make it possible to distinguish among different states of the system with different waiting times and to take in account the age t_0 of the material when it is subjected to failure processes. Furthermore, as demonstrated in Molina et al. (1996) and in Garavaglia et al. (2002), experimental results suggest that the deterioration process renews itself at every loss of surface layer. Therefore, it can be modeled as a renewal process (the simplest of s-MP) (Cox 1962).

Semi-Markov Process: Some Remarks

A s-MP process is defined when the following quantities are known (Howard 1971):

1. Initial conditions: the initial state, i.e., the state occupied by the system at the time of origin $t=0$; and time t_0, the length of time spent in the initial state at time $t=0$.
2. Probability density function $f_{ik}(t)$ of the waiting time τ_{ik}, i.e., the time spent in state i if the next state is k

$$f_{ik}(t) = \Pr\{t < \tau_{ik} \leq t + dt\} \tag{14}$$

   where t=time measured from entrance in state i.
3. Transition probability matrix, p_{ik} defined as

$$p_{ik} = \Pr\{\text{next state } k, \text{present state } i\} \tag{15}$$

A s-MP process can have different transition probabilities and waiting times for successive transitions that differ from those of its first transition. Therefore, the waiting time of the first transition is described by a PDF ${}_f f_{ik}(t)$ while time τ_i is the rv representing the waiting time in state i, (i.e., the lifetime in state i) between two successive transitions, whatever the next destination may be and is defined by the following PDF:

$$f_{\tau_i}(t) = \sum_k f_{ik}(t)p_{ik} \tag{16}$$

When 1, 2, and 3 are known, it is possible to compute the probability that the system will occupy state j at time t if it was in state i at time $t=0$ (Binda and Molina 1990). Therefore, in the semi-Markov hypothesis, decay prediction depends only on the transition probabilities, p_{ik}, waiting time PDF, $f_{ik}(t)$, and initial conditions. The problem is now that of a suitable choice for the PDF, $f_{ik}(t)$. In choosing $f_{ik}(t)$ we must pay attention to the failure rate function (Binda and Molina 1990)

$$\phi_{ik}(t)dt = \Pr\{t < \tau_{ik} \leq t + dt | \tau_{ik} > t\} \tag{17}$$

Since deterioration is a renewal process, the longer the waiting time τ_{ik}, spent by the system in state i, the higher the probability that transition to the next state k will occur in the next $(t+dt)$ time interval (Molina et al. 1996; Bekker 1999). Therefore, in this case as well distributions with the function $\phi_{ik}(t)$ increasing with t and tending to ∞ as $t \to \infty$ are needed. Experimental evidence and the behavior of $\phi_{ik}(t)$ suggest the use of a distribution of the Weibull family (6 and 12) in this approach as well.

Deterioration Process as Renewal Process

Molina has already shown that the material deterioration process seems to renew itself at every transition, and each transition is very close to the following ones, with the system often passing by state i directly into state k (with $k=i+1$) (Molina et al. 1996). This leads us to model the material deterioration process as a renewal process, the simplest of s-MP. Of course, the points 1, 2, and 3 must also be defined for the renewal process (see previous section) so how the amplitude of the service states (Garavaglia et al. 2002).

In a renewal process a given component of the system has the rth transition at time T_r (Cox 1962), where

$$T_r = \tau_1 + \tau_2 + \cdots + \tau_i + \cdots + \tau_r \tag{18}$$

and τ_i, independent rv, are the waiting times spent by the component in each state $i(i=1,\ldots,r)$. As a consequence, the probability density function $f_{\tau_i}(t)$ of τ_i describe the waiting time of the system in each state i.

If the independent rv $\{\tau_1,\tau_2,\ldots,\tau_i,\ldots\}$ are identically distributed, all with the same PDF $f_{\tau_i}(t)$, the process is called a (ordinary) renewal process. If, instead, the PDF ${}_f f_{\tau_i}(t)$, describing the waiting time characteristic of the first transition, differs from the

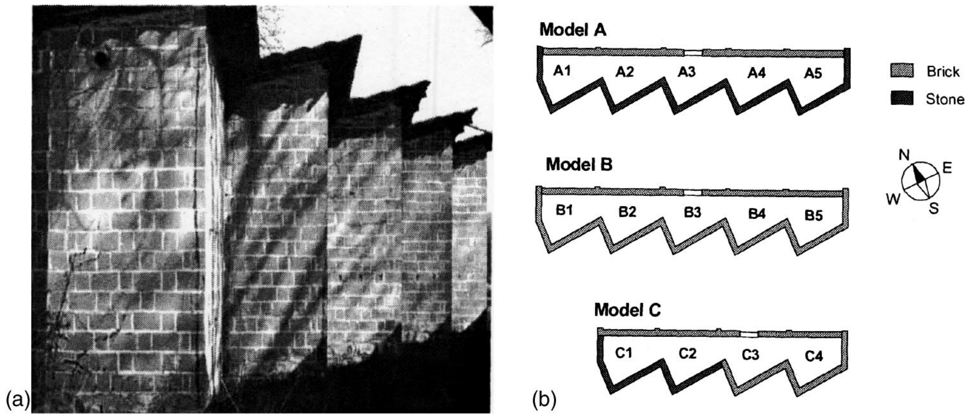

Fig. 2. Full scale outdoor specimens: (a) specimen B: stone panels and (b) design of specimens

PDF $f_{\tau_i}(t)$ (with $i>1$) describing the waiting times characteristic of each transition following the first, the process is called a modified renewal process (Cox 1962).

Therefore, calling

$$ {}_f f_{ik}(t) = {}_f f_\tau(t) \text{ (with } i = 0 \text{ and } k = 1) \tag{19} $$

the PDF of the waiting time describing the first transition and

$$ f_{ik}(t) = f_\tau(t) \text{ (with } i > 0 \text{ and } k > 1) \tag{20} $$

the PDF of the waiting times describing the transitions after the first one, the PDF $k_r(t)$ of waiting time T_r of the rth renewal of the whole system can be obtained by

$$ k_r^{**}(s) = {}_f f_\tau^{**}(s)\{f^{**}(s)\}^{(r-1)} \tag{21} $$

where $k_r^{**}(s)$=Laplace transform (LT) of the PDF $k_r(t)$

$$ k_r^{**}(s) = \int_0^\infty e^{-st} k_r(t)dt \tag{22} $$

where s=complex variable. An alternative notation for the Eq. (22) is $L\{k_r(t);s\} \equiv k_r^{**}(s)$. The PDF $k_r(t)$ can now be evaluated through the Inverse LT (ILT).

Fragility Curves versus Renewal Process: Some Results

Experimental Data

To record data on masonry decay due to environmental attacks, some full-scale specimens were built in 1990 in an industrial area close to Milan, Italy [Figs. 2(a and b)]. These models form a real outdoor experimental laboratory, coordinated by Binda and Baronio in collaboration with ICITE-CNR of S. Giuliano Milanese and ESEM of Milan, Italy (Binda et al. 1999). The geometry and exposure of the specimens were designed to obtain the fastest decay in a natural environment: their facades were composed of modular orthogonal panels exposed to the south and west [Fig. 2(b)]. The panels were made of sandstone and brick and putty lime mortar.

In an aggressive environment one of the most important causes of deterioration for the masonry is the salt crystallization. Binda et al. (1999) describe the laboratory test simulating this phenomenon. Salt crystallization produces high stress within the material; the effect is a continuous crumbling and delamination of the exterior surface of the wall, while the inside is left unaltered. For this reason the variation in roughness of the surface has been assumed as a measure of the damage to the masonry. The procedure used to measure the surface decay is based on laser device. The device draws a diagram of the wall roughness at selected scan sits (Fig. 3). Subsequent measurements show how the diagram changes in time due to any superficial decay. Using this procedure it is possible to measure the loss of material over time. The instrumental error of laser resulted to be less than 0.2%.

Fig. 4(a) is an example of the diagram for six different measurements taken from 1993 to 1998. The presence of swelling was observed. Since bulging (or swelling) the step before delamination, it can be considered as the starting point of damage. The experimental measurements need to be converted into new deterioration diagrams where the bulging has been eliminated; this has permitted to quantify correctly the swelled material as a "lose." The procedure compares the horizontal coordinates of two subsequent diagrams: the current plot n and the previous plot $(n-1)$. The coordinates of the diagram n are usually smaller than these of the diagram $(n-1)$, except for the points affected by swelling. In these points the computer code continues the procedure, comparing the coordinates of plot n with these of the successive plots $(n+1)$, $(n+2)$, $(n+3)$, … until a plot $m=(n+i)$ having smaller horizontal values than these of profile n is found. The coordinates of diagram m, corresponding to the points affected by bulging, become the new reference coordinates of profile n when is redrawn. As a result, a clean plot of the evolution of the surface damage as a function of time and space is obtained [Fig. 4(b)] (Garavaglia et al. 2002).

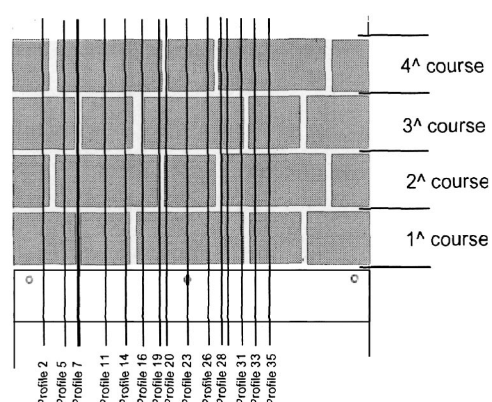

Fig. 3. Positions of profile measurements of wall

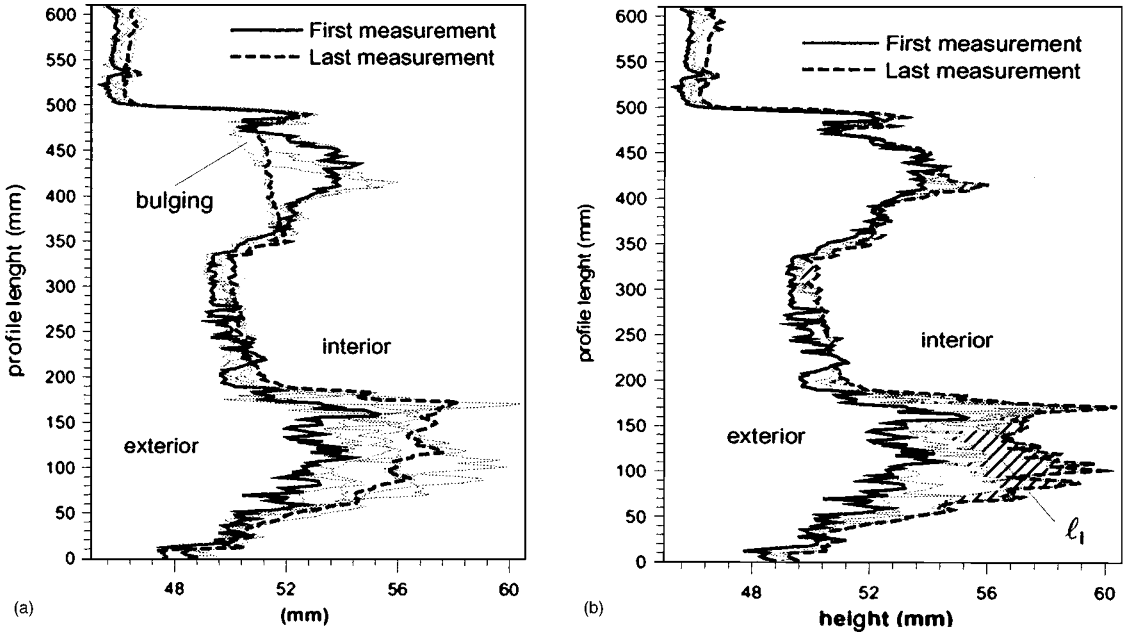

Fig. 4. Measurement of deterioration overtime before (a) and after (b) swelling has been removed (profile number 7)

The loss of material seems to be a good parameter for quantifying the masonry damage due to salt crystallization and it can be quantified using the new diagrams of Fig. 4(b). Therefore, for each profile i, represented in Fig. 4(b), the loss l_i of cross section of the wall (in millimeters square), calculated at every time t^* of measurement ($t^*=6$, 17, 22, 44, and 60 months), has been assumed as the parameter of damage for decay due to salt crystallization. To quantify l_i, the area included between two consecutive diagrams is calculated at every time t^* [Fig. 4(b)]. This area is automatically calculated by the computer code studied to eliminate bulging by means of the composite trapezoidal rule (Garavaglia et al. 2002).

In Fig. 5 the plot of the damage l_i versus time is reported for each profile i. A simple interpolation of the experimental point permits a better reading of the behavior of the loss over time (linear splines).

Following the fragility curves approach for a given time t^*, the deterioration can be modeled through a log-normal PDF Eq. (2) (Fig. 5), and the resulting experimental fragility curves evaluated (Fig. 6). The shape parameters α and ρ involved in the distribution Eq. (6) have been estimated through a computer code using the maximum likelihood method and Rosenbrock's optimization method. This estimation approach has been used, with good results, to evaluate the parameters of all the distribution used here below. This modeling can also be obtained through the function *FMIN* present in the *MATLAB* code.

Results Obtained by Application of Two Approaches

The probabilistic approaches proposed here are able to model the deterioration process of the masonry components in terms of the probability of reaching or exceeding a given damage threshold $\bar{l}$ over time.

Through the fragility curves obtained by computing Eq. (6), it is possible to identify the failures with a higher or lower probability of occurrence in the given time t (Fig. 6). For example, in Fig. 6 it is evident that the probability of exceeding a given damage $\bar{l}$ in a short time is lower when $\bar{l}>600$ mm^2 and higher when $\bar{l}<400$ mm^2. The application of this approach is simpler than the application of a semi-Markov approach, but requires a very long monitoring time for significant results, meaning a mass of recorded data as well.

If the deterioration process is a renewal process, the probability of reaching or exceeding different damage thresholds $\bar{l}$ over time t can also be calculated through the LT and its inverse (ILT).

The application of the semi-Markov approach is not simple. As said above, it requires the definition of some service states through which the system passes during its deterioration process (each state is characteristic of a given damage level). The choice of these service states is not simple, and being arbitrary may compromise the interpretation of the analyzed process. In fact it depends on the type of failure and on the physical aspect of the investigated phenomenon. As said in a previous section, Molina et al. (1996) have shown that the salt decay process of

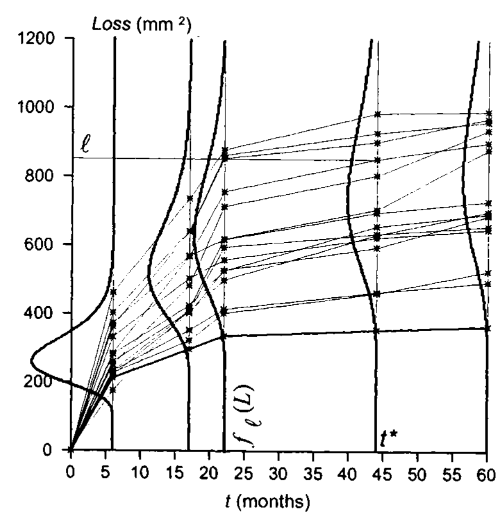

Fig. 5. Interpolation of loss diagrams (*) and modeling of deterioration process $L(t,l)$

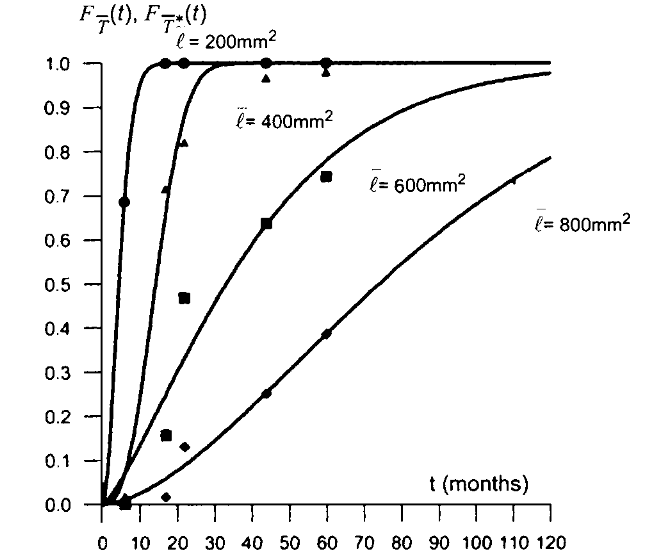

Fig. 6. Fragility curves for different $\bar{l}$: experimental $F_{\overline{T}^*}(t)$ (symbols), theoretical $F_{\overline{T}}(t)$ (—)

masonry materials can be modeled as a renewal process. Here the choice of the state has been finalized to verify whether the decay process of masonry as a composite material is still a renewal process, consequently, the choice was not so difficult.

In the case studied the experimental evidence has shown that it is possible to distinguish:

1. an initial state, state 0, with $t_0=0$: as the wall was new when the test started, the elapsed time in the state i at the initial time $t=0$ is 0;
2. the first waiting time, representative of the first delamination and modeled by $_f f_\tau(t)$, with a *first* loss equal to 400 mm²; and
3. the successive waiting times τ, representative of the following transitions and modeled by $f_\tau(t)$: the losses following the first one are equal to 200 mm².

Such close state thresholds guarantee a sure transition between the different states; in this case the transition probability is: $p_{ik}=1$.

As dated above, another important point is the choice of the densities involved in the renewal process. This depends on our comprehension of the physical phenomenon investigated as does the choice of the service states.

The experimental data have demonstrated a bimodal behavior for both $_f f_\tau(t)$ and $f_\tau(t)$, which have, therefore, been modeled with a mixture

$$f_\tau(t) = p w_1(t) + (1-p) w_2(t) \tag{23}$$

where $w_1(t)$, a PDF, of small average value, indicates a short term damage; $w_2(t)$, a PDF of higher average value, indicates a long term damage; while p and $(1-p)$=, respectively, that of a close first transition and the probability of a delayed first transition. For the reasons given above, the Weibull distributions $w_1(t)$ and $w_2(t)$ have been chosen.

Figs. 7 and 8 compare the cumulative distribution of test data and the theoretical distribution (a mixture of the two Weibull distributions), both the $_f F_\tau(t)$ modeling the first transition and the $F_\tau(t)$ modeling the transitions after the first. The goodness of the fitting is evident.

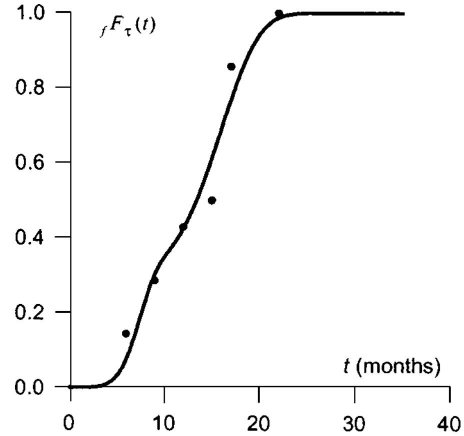

Fig. 7. First waiting time $_f\tau$ comparison between cumulative distribution of test data (●) and curve fit by two Weibull distributions (—)

Comparison Between Two Proposed Approaches

Fig. 9 compares the two approaches. The results are clearly similar. The renewal process seems to be more conservative than the fragility curves on long-term prediction, but the differences recorded are not significant. Therefore, the choice of the approach to use depends *only* on the available data and the monitoring time. It is important to note that the renewal approach allows prediction of exceeding thresholds even where, at the moment, no experimental information is available (e.g.: $\bar{l}=1{,}000$ mm² in Fig. 9).

The advantage of the fragility curves approach is that it is simple and does not require any particular basic assumptions. But to have significant results the time of monitoring and data recording has to be very long. The renewal process instead, takes into account the time t_0 elapsed since the construction of the building, or the surface treatment and the instant when the analysis of its deterioration starts. Therefore, no information is lost. Moreover, if the process modeled is a renewal process the time of monitoring can be reduced because the process renews itself at every $\bar{l}$ of material lost; so only the knowledge of the waiting times describing the first two transitions is needed to completely define the long-term process.

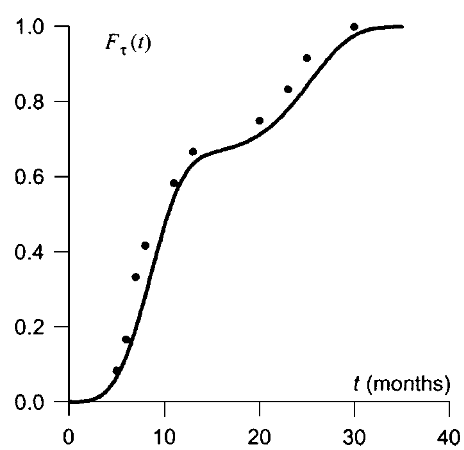

Fig. 8. Waiting time τ comparison between cumulative distribution of test data (●) and curve fit by two Weibull distributions (—)

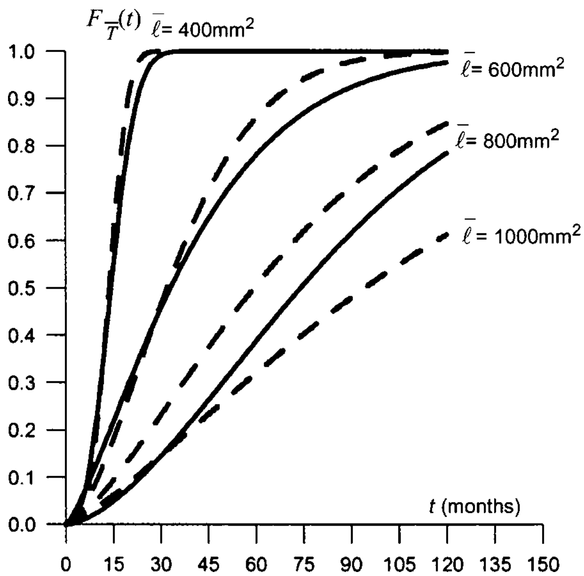

Fig. 9. Comparison between two approaches: fragility curves (—); renewel process (---)

Conclusions

The approaches proposed to model the deterioration process provide information, in probabilistic terms, on the occurrence time of a given damage level $\bar{l}$ to the system studied. By computing the fragility curves the probability exceeding a given damage level $\bar{l}$ at every time t is defined, so that the probability of occurrence of different damage levels at the time t^*, can be easily evaluated. The renewal process can predict, in probabilistic terms:

- the occurrence time of the r^{th} renewal; and
- the number of renewals in a certain time t^*. Since a renewal occurs every time the damage threshold $\bar{l}$ is reached, this approach also allows predictions of:
- the occurrence time of the damage level L_r connected with the r^{th} renewal; and
- the level of damage reached with the renewals occurring over time t^*.

The knowledge of the probability of occurrence of a given damage level $\bar{l}$ at the time t, means that it is possible to plan maintenance strategies known in terms of execution, durability and effectiveness. For example, the time t^*, connected with the probability of exceeding $\bar{l}$, can be assumed to be the right moment to perform maintenance on the system. This will leave the system in a better service life than that corresponding to time t^*, although it is usually unable to restore the system to its original (first) service state, that is, a certain damage may remain. Further analysis is needed to take into account this damage, so the probability of exceeding $\bar{l}$ could be reached in a time $\hat{t} < t^*$, and the time interval between the maintenance actions is never constant.

The use of the proposed approach allows the investigation of this eventuality. Following the same methodology, questions concerning the durability and the effectiveness of maintenance and repair, as well as the surface treatment can be dealt with.

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Notation

The following symbols are used in this paper:

b_1, b_2 = constant;
e = 2.718281828;
$\mathcal{F}_l$ = survive function of l;
$F_{\bar{T}}$ = cumulative distribution function of $\bar{T}$;
$F_{\bar{T}^*}$ = experimental fragility curve;
F_τ = comulative distribution function of τ;
F_{τ_i} = comulative distribution function of τ_i;
f_{ik} = probability density function of τ_{ik};
f_l = probability density function of l;
$f_{\bar{T}}$ = probability density function of $\bar{T}$
${}_fF_\tau$ = cumulative distribution function of τ describing first transition;
${}_ff_{ik}$ = probability density function of τ_{ik} describing first transition;
k_r = probability density function of T_r
$L(t)$ = deterioration process over time;
$\bar{l}$ = damage threshold;
l = loss of surface material;
p_{ik} = probability of transition;
$\bar{R}$ = probability to exceed threshold $\bar{l}$;
R = reliability function;
s = complex variable;
$\bar{T}$ = time to exceed $\bar{l}$;
T_r = time or rth transition;
t = time;
t_0 = age of material;
t^* = time of experimental measurement;
w = Weibull distribution (α=shape parameter, ρ=scale parameter)
α, ρ = parameters of Weibull distributions;
η, σ = parameters of lognormal distributions;
τ = lifetime;
τ_i = waiting time spent in state i;
τ_{ik} = waiting time for transition from state i to state k;
ϕ_{ik} = failure rate function of τ_{ik};
ϕ_l = failure rate function of l;
$\phi_{\bar{T}}$ = failure rate function of $\bar{T}$; and
$**$ = Laplace transform symbol.

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