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The credibility of lifetime assessment based on few experimental measurements

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ABSTRACT: Recently an index of credibility has been introduced by Grandori et al. (2003) in order to measure the variability of an estimated hazard quantity. On its basis a probabilistic hazard model can be constructed reducing some aleatory uncertainties. In this paper the definition of the credibility index is extended to the lifetime assessment of reinforced concrete structures affected by deterioration over time.

1 INTRODUCTION

When a hazard quantity *a* concerning rare events has to be estimated, the estimated value $\hat{a}$ is affected by a very large uncertainty. The background hypothesis cannot be validated in an *absolute* sense: the classic statistical analysis assigns equivalent degree of acceptance to different hypotheses and no direct quantitative evidence can be derived from the poor data set.

Here a *relative* cryterium of validation is used based on an estimator that measures the variability of estimated quantity; two competing models are taken in consideration and judged on the basis of the above index.

The preference of one model is decided in respect to a fan of conjectural realities.

In this paper the definition of this index, called *credibility index* is extended to the lifetime assessment of reinforced concrete structures affected by deterioration over time. In it, it is dealt the prevision of the behaviour, over time, of a damage quantity when few direct measurements are available and investigate the credibility of this prediction. The aim of this approach is to give information on the time of maintenance in presence of few monitoring measurements.

2 THE PROBLEM OF UNCERTAIN IN PREDICTION

Two kinds of uncertainties reside in a probabilistic model: epistemic and aleatory (or statistic). Epistemic uncertainty is concerning the formal model: it can be wrong. Even if the model is correct, another uncertainty subsists because the model contains parameters which must be estimated from the available data. But the available sample of data is only one among all performable samples. So the set of estimated parameters is only one among all possible sets. This is the concern of statistic uncertainty.

In structural analysis the available samples are the monitoring results. Usually the monitoring of a given significant parameter is made along the life-time of the structure in discrete instants; only at the end of a long monitoring will be possible to describe the variability of the parameter over time. Instead it is interesting to predict the parameters variation law over time on basis of few monitoring data. Therefore, the prediction must be made in statistic or probabilistic terms, but the questions are: "*which is the true model?*", "*which are the true parameters?*".

These questions cannot receive answer. The questions are wrong questions. Unfortunately even what is called "statistical validation" is not reachable. To be fully statistically substantiated, a proposed model "should be backed by a sufficient number of directly observed successes and failures to establish its performance at some agreed level" (Vere – Jones 1995).

About the term "validation" largely in use in the last years, it should be noted that never a probabilistic model can be validate in absolute sense (Grandori et al. 1998, 2003).

A probabilistic model consists in a continuous way towards more robust degree of knowledge (Lakatos et al. 1974). The Bayesian approach teaches how, starting from a basic guess, a basic prior probability, our confidence grows towards a posterior more credible probability when more information becomes available and can be incorporated in the probabilistic model.

Usually, in statistical analysis the chosen between different models can be supported by the classic statistical tests if other evaluations are done, typically: comparison between models and methods with relative likelihood functions or least squares method. They help to explore the reliability of one model (or hypothesis). Particular care deserves the underlying physics of the hazard: a "model" is not a pure statistic consequence of a data set. Each distribution contains some physical interpretations. So, in the case of bending moment behavior, the physical knowledge about the mechanism of damage can be incorporated: typically crack propagation, carbonation, corrosion, diffusive attacks (Guagenti et al. 2003, Ardigò et al. 2002, Biondini et al. 2004).

In conclusion a (probabilistic) model can be supported by a wide patient data interpretation in such a way as to get a certain degree of confidence; even if it contains always a subjective choice, a careful analysis makes reasonable the choice. Obviously the most unsatisfactory residual field of uncertainty remains in problems concerning small samples. When different independent groups of experts are asked to evaluate the same quantity a the dispersion of the results is, in general, very large. Some years ago, an index Δ called credibility has been introduced (Grandori et al. 1998, 2003), in order to reduce, this dispersion. Grandori et al. Shown like the index Δ has the merit to focus the possible error in the quantity a estimation and, this way, to decide between two candidate models which one is the winner in the estimation of a.

This index has been developed to investigate the peak ground acceleration (PGA) $a(500)$ at a given site, corresponding to 500 years return period, which is typically assumed as a measure of the seismic hazard for engineering purposes: it is prescribed in the code as that acceleration which a normal building must resist to, without damage. *Which is a reliable estimate* $\hat{a}$ *of* $a(500)$? Note that such a PGA may have never occurred at the site in the period of observation. Nevertheless, the code requires an estimate of this value. The quantity $a(500)$ is a long term static estimate of average risk (here the term "risk" is used as synonymous of "hazard"). For this estimate a static model is reliable. In this problem the hazard can be considered constant in time.

On the basis of this approach, it has been developed the prediction of the acting bending moment in a reinforced concrete structure at the time t^*, lager than the last monitoring time, starting form a single sample and few monitoring data.

3 AN IMPROVEMENT IN VALIDATION

We give here an outline of the announced effort towards a validation of a model. Model here is essentially a *paradigm* to estimate a quantity m starting from a single data set. In fact the substantial difference with the standard statistical analysis consists in shifting the attention from the data fitting to the error in estimating the quantity m of interest.

Beyond discussion an absolute validation; more modestly we limit ourselves to the comparison between two models in competition to estimate a quantity m.

Which one of the two estimates coming from the two models is more reliable? This is the question. In this paragraph we apply the method proposed by Grandori at a typical structural problem: the life-time prediction.

If we suppose that in a given R.C. structure the time evolution of a given parameter is completely defined, we can suppose that also our "truth" is completely defined and the quantity M assumes a precise value M° at the time t^*. In other hands, if the variation law $F_M(t)$ of a given quantity M is given, both like form and parameters, the value assumed by M in each instant t^* is known.

From now on, a distribution with known parameters will be briefly indicated with an upper index; on the contrary lower index will indicate that estimable parameters are present in F.
The distribution F^o being known, it is possible (with Monte Carlo simulation) to draw many size ν samples from it (different conjectural realities). Starting from each sample our model (i.e. an F_r) leads to estimated values $\hat{M}_r$, one for each sample. All together they form the random variable $\hat{M}_r$, whose distribution is the sampling distribution of the bending moments.
The index:

$$\Delta_r^o = \Pr\left\{ M^o - h < \hat{M}_r \leq M^o + h \right\} \quad (1)$$

is the probability that the estimated value $\hat{M}_r$ with the r – model falls in a given interval around the "true" value M^o (h defines a conventional interval around M^o). The index Δ_r^o takes into account both *epistemic* and *aleatory* uncertainty; it measures the *credibility* of r – model in respect to F^o when M is the estimable quantity (credibility not based on data fitting, but looking at the result).

Briefly: Δ_r^o is the probability that, on the basis of a random size ν sample drawn from F^o, the model F_r leads to estimate M with an error $\varepsilon_r^o \leq h$ (as absolute value)

$$\Delta_r^o = \Pr\left\{ \left| \varepsilon_r^o \right| \leq h \right\}. \quad (2)$$

Analogously for the second model s – model (i.e. for an F_s):

$$\Delta_s^o = \Pr\left\{ M^o - h < \hat{M}_s \leq M^o + h \right\}. \quad (3)$$

The difference

$$\Delta_{rs}^o = \Delta_r^o - \Delta_s^o \quad (4)$$

is a meaningful index of the *relative credibility* of the two models. We assume that r – model is more reliable than s – model for the estimate of a if $\Delta_{rs}^o > 0$ and viceversa. Then the sign of Δ is of critical importance: it decides which model is the winner.

What is the interest given that the true M° is not known?

We can start operating on the unique object available: the data-set obtained by a monitoring action, limited in time, operated on a given structure. By these data we construct an empirical truth furnished by the law F^* (usually a polyline) and, proceeding as above described, we construct the empirical index Δ_{rs}^*. The question now is: how much this empirical index is *representative* of the true unknown Δ_{rs}^o?

The degree of relationship is well represented by the two conditional probabilities:

$$^{\downarrow}\mathrm{H} = \Pr\left\{ \Delta_{rs}^* > 0 \;\middle|\; \Delta_{rs}^o = x > 0 \right\}. \quad (5)$$

$$^{\uparrow}\Omega = \Pr\left\{ \Delta_{rs}^o > 0 \;\middle|\; \Delta_{rs}^* = x > 0 \right\}. \quad (6)$$

Equation 5 is a probability *"towards down"*: given the hypothesis that Δ^o is positive calculate the probability to observe Δ^* positive. So a first degree of agreement with the hypothesis is obtained.

Equation 6 is the posterior probability Ω which is a probability "*towards up*": given the ob-

served data, evaluate the probability of hypothesis. In precise terms to calculate Ω we should know all the possible realities. So, in precise term Ω can not be calculated. But with patience we can explore a lot of plausible realities. For each of them we can repeat the simulations and the estimate above described.
The technical details are given in Grandori et al. present in reference list. So, the observed value Δ^* becomes much more than a symptom: it can measure the probability that one model is winner against the other one.

4 APPLICATION OF PROCURE AT A STRUCTURAL PROBLEM

Let us try to extend the definition of the credibility index Δ to the case of bending moment prediction. Usually, during the first 20 years of the structure few measurements of bending moment can be made, but interesting is to study its evolution law over time on basis of few data. In this case the model to be judged consists in the evolution law F_{M_S} of the acting bending moment M_S(t) in a R.C. structure obtained throughout few measurements. Therefore, to extend the index Δ to the present case the quantity to estimate here is a "*value*": precisely the acting bending moment $M_S(t)$ with t=50years assumed as a critical instant of the lifetime of the structure.

By experimental evidence the law describing the evolution of M_S can be assumed to be a polynomial function or a linear function, however the choice between the two laws is not foreseen.

In the case under investigation, the "*error*" is the "difference" between the value of $\hat{M}_{S_r}$ estimated with the r – model and the "true" value M_S^o. Therefore the index assumes the following form:

$$\Delta_r^o = \Pr\left\{(1-h)\, M_S^o \le \hat{M}_{S_r} \le (1+h)\, M_S^o\right\}, \tag{7}$$

where h defines a conventional interval around M_S^o (the ^ is the mark of functions or quantities derived from the samples).

Briefly: Δ_r^o is the probability that, on the basis of a random size-ν sample drawn from $F_{M_S}^o$, the model F_r leads to estimate M_S with an error $\le h$.

In the case of two competing models F_r and F_s, we call "relative credibility index" the difference

$$\Delta_{rs}^o = \Delta_r^o - \Delta_s^o \ , \tag{8}$$

and we assume that F_r is more credible than F_s for the estimate of the quantity M_S if Δ_{rs}^o >0 and viceversa; this means that, compared with F_s, the model F_r leads to a value of M_S "closer to M_S^o" in probabilistic terms.

Now the question is: can the definition of Δ_{rs}^o be useful given that the real $F_{M_S}^o$ is not known? Well, from the real sample of data actually available it is possible to describe the law $F_{M_S}^*$ (the * is the mark of functions or quantities obtained from the real sample) and, as consequence, the $\hat{M}_S^*$. The following step is the evaluation of Δ_{rs}^* and the check of how much the value of Δ_{rs}^* is representative of the sign of Δ_{rs}^o, which is the discriminating element of comparison.

5 FIRST RESULTS

The procedure proposed is here applied to predict the life time of a continuous reinforced concrete T-beam subjected to aggressive environmental attacks using few monitoring data. To investigate the bearing capability over time, the evolution of the acting bending moment M_S is considered. The same problem has been dealt by Ardingò et al. (2002) and by Biondini et al. (2004) through a cellular automata approach to durability analysis of R.C. structures in aggressive environment; for the structure of the approach used by Biondini, the results obtained can be considered as “statistical truth” and here used like a comparison with the results obtained by the approach proposed.

5.1 *Description of structure*

The R.C. continuos T-beam presented in Figure 1a, has a span lengths L=3.00m and load g=10 kN/m. The cross-section has the geometry shown in Figure 1b, main dimensions are H=0.40m, h=0.25m, B=0.40m, and b=0.15m.

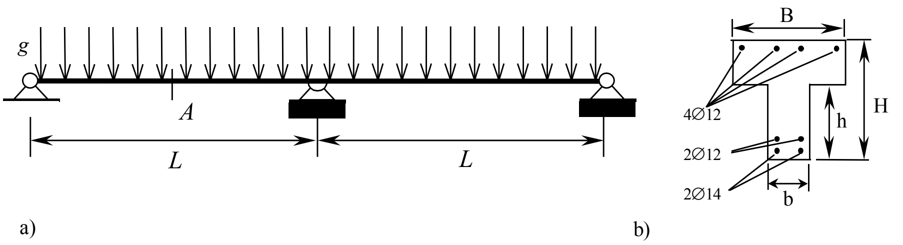


Figure 1. Reinforced continuous beam: a) structural model and load condition; b) geometry of the cross-section and location of the reinforcement.

The behavior of concrete is linear elastic with modulus E_c=35 GPa in compression and with no strength in tension. For steel, a linear elastic behavior with modulus E_s=206 GPa is assumed. The beam has been assumed having a deterioration along the time. The coupling effects between diffusion process and the cracking state leads to a non uniform damage along the beam and the bending moment diagram evolves over time (Biondini et al. 2004); in particular Figure 2 make a comparison between the evolution of the acting bending moment M_S and the corresponding resistant bending moment M_R in section A (results obtained by a non linear analysis).

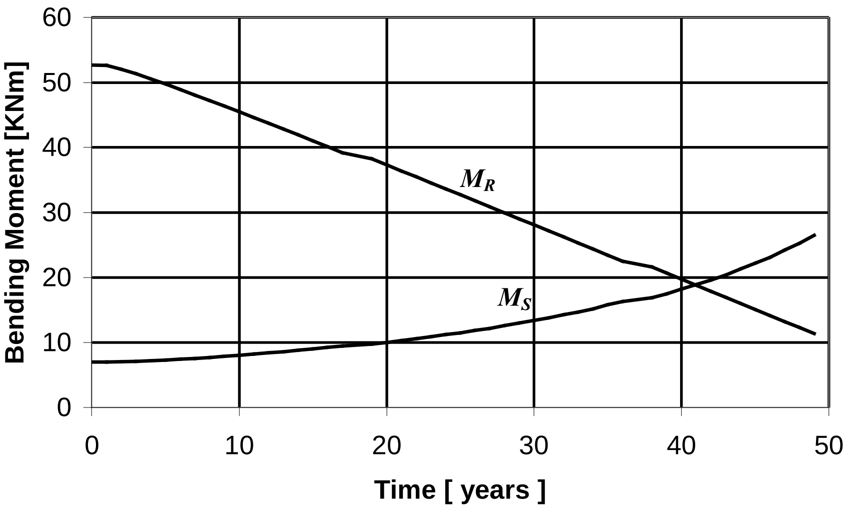
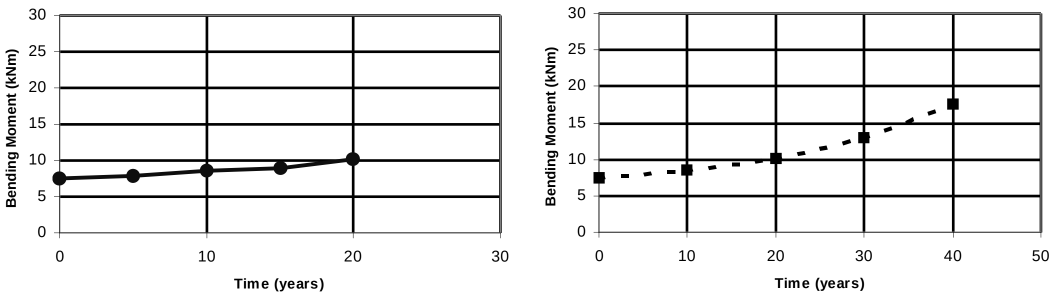


Figure 2. Comparison between the time evolution of both the acting M_S and resistant M_R bending moments in cross section A (adapted from Biondini et al. 2004, present in reference list).

5.1.1 *Monitoring data*

After a monitoring investigation of 5 measurements on the beam along 20 years, the M_S evolution can be described like shown in Figure 3a. This information can be ambiguous: by experience the law evolution of M_S can be linear or polynomial. It is clear that prediction of M_S at a given time can suffer by uncertainty. To reduce this uncertainty necessary is to extend the monitoring time (Fig. 3b) or to try to understand the tendency of the law already from few monitoring. In the follows will be shown as the credibility index Δ_{rs} can lead to comprehend the tendency of behaviour and to plan a suitable monitoring action.



a) b)

Figure 3. Experimental measurements of acting M_S bending moment in cross section A: a) t=20years, b) t=40years

5.1.2 *Hypothesis: the evolution law of M_S is known*

As said above, Equation 8 is able to indicate the winner between two models in competition if $F^o_{M_S}$ is known. To test the approach proposed the two possible law already discussed (linear and polynomial) are take in consideration. From each law 1000 samples of 5 $\hat{M}_S$ values are been obtained with a random procedure over an arch time of 20 years (conjectural reality) and 40 years (always a conjectural reality); following the Monte Carlo simulation procure each sample as been modeled with both the model r and the model s and $\hat{M}_S(50)$ at fifty years has been evaluated. The following step has been the evaluation of Δ^o_r, Δ^o_s and Δ^o_{rs} has been made using h = [0.05 x $M^o_S(50)$]. The results obtained by this procedure are reported in Table 1 for the law $F^o_{M_S}$ *linear* and Table 2 for the law $F^o_{M_S}$ *polynomial* of 2[nd] degree.

Table 1. *Credibility Index* evaluation: *Law:* linear, *model r* = linear, *model s* = polynomial.

Time of monitoring	Δ^o_r	Δ^o_s	Δ^o_{rs}	$M^o_{S_r}(50)$	$\bar{e}_{rr}$	$\bar{e}_{rs}$
20 years	0.578	0.086	0.492	11.75	0.552	3.32
40 years	0.859	0.217	0.642	11.75	0.537	1.32

Table 2. *Credibility Index* evaluation: *Law:* polynomial, *model r* = linear, *model s* = polynomial.

Time of monitoring	Δ^o_r	Δ^o_s	Δ^o_{rs}	$M^o_{S_s}(50)$	$\bar{e}_{ss}$	$\bar{e}_{rs}$
20 years	1E-04	0.070	-0.06990	42.00	0.39	20.70
40 years	0.00112	0.403	-0.40188	42.00	1.43	8.19

By results shown in Table 1 and 2 it is emerged that the credibility index Δ^o_{rs} is a good indicator to catch the tendency of the parameter at a given behaviour also on the basis of few measurements. It is able to distinguish the winner between two model in competition: in fact when the law is *linear* $\Delta^o_{rs} > 0$ it means that the model *r* is the winner and the model *r* is "*linear*"; when the law is *polynomial* $\Delta^o_{rs} > 0$ it means that the model *s* is the winner and the model *s* is "*polynomial*". Of course the credibility increase if the time of observation increase. In the case of polynomial law Δ^o_{rs} at 20 years present a value close to zero, this could lead to say that there is no difference modelling the data with *r* or *s* but the "symptom": the sign of Δ^o_{rs}, is of a polynomial behaviour (Δ^o_{rs}>0), confirmed with a longer time of monitoring. As comparison, a typical statistical error *e* as been built with the least squares method; the average value $\bar{e}$ seem to confirm the decision: $\bar{e}_{ss} << \bar{e}_{rs}$ means a tendency to polynomial behaviour.

5.1.3 *Realty: the evolution law of M_S is unknown*

In the common practice, the evolution law $F^o_{M_S}$ of $M^o_S(50)$ is not known, therefore $F^*_{M_S}$, $\hat{M}^*_S(50)$ and Δ^*_{rs} must be *evaluated* in experimental form throughout a given procedure $\Delta^*_{rs} = E_{rs}(F^*_{M_s})$ (E_{rs} is the procedure chosen). In the case studied a discrete polyline law $F^*_{M_S}$ has been chosen and to investigate the value of $\hat{M}^*_S(50)$ with a simple increasing linear branch adopted to describe the unknown tendency of the law in the intervals [20years, 50years] and [40years, 50years]. It has been built starting from the behaviour of the previous segments of polygonal law (Figure 4). The results obtained are present in Table 3.

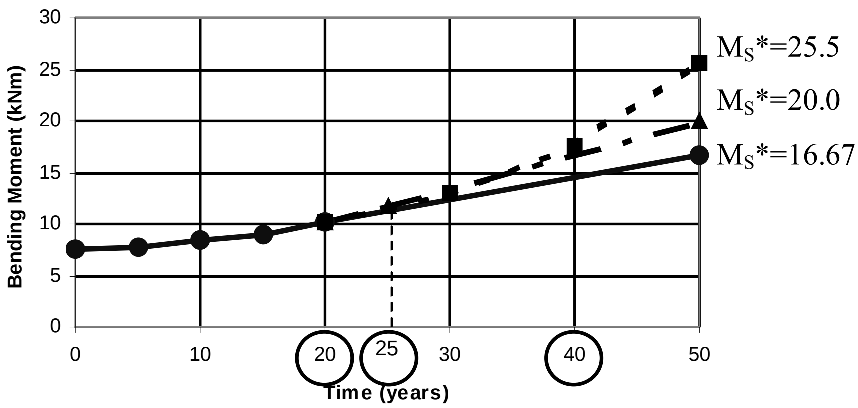


Figure 4. Behavior of experimental data: maximum monitoring time 20, 25 and 40 years and evaluation of $\hat{M}^*_S(50)$ throughout E_{rs} procedure.

Table 3. *Credibility Index* evaluation: *Law:* polyline, *model r* = linear, *model s* = polynomial..

Time of monitoring	Δ^*_r	Δ^*_s	Δ^*_{rs}	$\hat{M}^*_S(50)$	$\bar{e}_r$	$\bar{e}_s$
20 years	0.295	0.015	0.280	16.67	0.750	1.870
40 years	1E-06	0.406	-0.406	25.55	0.960	0.809
25 years	1E-07	0.058	-0.058	20.06	0.787	0.962

The results in Table 3 shown the difficult to interpret the phenomenon investigate if a time of monitoring too short is used. In fact, Δ^*_{rs} at 20 years is positive; it means that the model *r* is the winner with a $\hat{M}^*_{S_r}(50) = 16.67$, but at 40 years the tendency is changed: $\Delta^*_{rs} < 0$ means that the winner is the model *s* with a $\hat{M}^*_{S_s}(50) = 25.55$. It is clear that a wrong choice lead at a underestimation or an overestimation of $\hat{M}_S^*(50)$. Also the average value $\bar{e}$ of the error e is not able to improve the decision at 20 years; it is clear that a longest monitoring time is needed, for example a monitoring time of 25 years can already improve the decision (Table 3): the tendency to polynomial is declared by Δ_{rs}^*, but not by $\bar{e}$, this underline the goodness of the credibility index on $\bar{e}$ in prediction.

In Figure 2 (Biondini et al. 2004) the "true" value is $\hat{M}_S^*(50) = 28$ and the behaviour has a polynomial tendency; by a comparison with the "statistical truth" the results obtained confirm the goodness of the methods: Δ_{rs}^* is negative for both 20 years and 40 years analyses (Table 4).

Table 4. *Credibility Index* evaluation: *Law:* "statistical truth", *model r* = linear, *model s* = polynomial.

Time of monitoring	Δ_r^*	Δ_s^*	Δ_{rs}^*	$\hat{M}_S^*(50)$	$\bar{e}_r$	$\bar{e}_s$
20 years	0	0.033	-0.033	28	0.446	0.168
40 years	0	0.075	-0.075	28	0.275	0.125

6 CONCLUSIONS

The index Δ, successfully introduced as credibility of a probabilistic model in evaluating a static hazard quantity, as been here proposed as a possible validation index in the prediction of the life-cycle of a structural element. The results obtained, compared also with a "statistical truth" has shown the goodness of procedure also to indicate an adequate time length of monitoring finalized to obtain reliable news on the behavior of the phenomenon investigate.

7 REFERENCES

Ardigò, C., Biondini, F. & Malerba, P.G., 2002. A Cellular Automata Finite Beam Element for Damage Evaluation and Durability Analysis of Concrete Structures, Proceedings of the Second International Conference on Advances in Structural Engineering and Mechanics, *Proceeding of* ASEM'02, Pusan, Korea, 21-23 August 2002.

Biondini, F., Bontempi, F., Frangopol, D.M & Malerba P.G. 2004. Cellular automata approach to durability analysis of concrete structures in aggressive environment. ASCE Journal of Structural Engineering – (tentatively accepted for publication).

Grandori, G., Guagenti, E. & Tagliani, A., 1998. A proposal for comparing the reliabilities of alternative seismic hazard models, *Journal of Seismology,* 2, 27-35, 1998.

Grandori, G., Guagenti, E. & Tagliani A., 2003. Magnitude distribution versus local seismic hazard, BSSA, *Bull. of Seismological Society of America* , June 2003, Vol.93, No.3, pp 1091-1098.

Guagenti E., Garavaglia, E. & Petrini L., 2003. Probabilistic Hazard Models: is it Possible a Statistical Validation? in: "System – based Vision for Strategic and Creative Design" Proc. of ISEC02, 2nd Int. Structural Engineering and Construction Conf, Rome, Italy, U.E., September 23 – 26, 2003, F. Bontempi Ed., A.A. Balkema Publishers, Lisse, The Netherlands, Vol. II, pp. 1211 – 1216.

Lakatos, I. & Musgrave, A., 1974. *Criticism and growth of knowledge*, Cambridge University Press.

Vere-Jones, D., Harte, D. & Kosuch, M., 1998. Operational requirements for an earthquake forecasting programme in New Zealand, *Bull. of the New Zealand National Society for Earthquake Engineering*, 31, 194-205.