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Uncertainties in the nonlinear analysis of masonry bridge structures

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ABSTRACT: The paper deals with the analysis of the uncertainties related to the structural response of bridges composed by blocks of stone. The main feature of this class of bridges is the presence of two distinct materials, the stone and the mortar, having different stiffness and strength properties. The uncertainties involved in the formulation of the problem are modeled by using a fuzzy criterion and the structural response is studied by taking the nonlinear behavior of the system into account. Therefore, the main item of the paper is to show the effectiveness of the fuzzy nonlinear analysis in typical engineering problems having practical consequences.

1 INTRODUCTION

For structural systems having non-linear behavior, like the masonry bridges, a realistic description of the response under all load levels can be obtained only by taking the actual non linearity into account. Moreover, the geometrical and mechanical properties which define the structural problem cannot be usually considered as deterministic quantities. In the analysis of masonry bridges the uncertainties can involve several quantities like the load histories, the geometry and the topology of the structure, the dimension of the blocks (brick or stone), the thickness of the joints, the dimension of eventual mortar beds, the material properties of the stone and of the mortar.

In this context, thought the reliability of the structure as resulting from a general and comprehensive examination of all its possible failure modes, one must pay attention to the following four aspects which define the assessment process:

1. Available data.
2. Modeling of the uncertainties.
3. Nonlinear structural analysis.
4. Synthesis of the results.

In this paper the uncertainties connected to the mechanical behavior of bridges composed by blocks of stone are modeled by using a fuzzy criterion (Biondini, et al.,.2000). The main feature of this class of bridges is the presence of two distinct materials (stone/mortar) whose stiffness and strength properties are so different that one can reasonably assume the deformations to be fully located in the mortar sections, which are then expected to be critical points for the overall equilibrium (Livesley, 1978). Based on such hypothesis, a finite element able to model this class of masonry structures is presented. The stone is supposed to be a linear material, whereas the mortar is modeled as a no-tension material having nonlinear behavior in compression and plastic frictional sliding according to a Coulomb law. The main uncertainties of the problem are usually related to the parameters which define the material laws and the thickness of the mortar layers.

The proposed approach has been applied to the analysis of a masonry arch bridge and the results are used to show the effectiveness of the fuzzy nonlinear analysis in typical engineering problems having practical consequences.

2 SEARCH FOR THE RESPONSE INTERVAL AS AN OPTIMIZATION PROBLEM

Let *p* be a parameter belonging to the set of quantities which define the structural problem and λ a load multiplier. It is clear that to each set of parameters corresponds a set of limit load multiplier, one of them for each assigned limit state (Livesley, 1978, 1992). For sake of simplicity, we start our considerations by considering the relationship between one single parameter *p* and one single limit state defined by its corresponding limit load multiplier λ. At first, it is worth noting that, in general, such relationship is nonlinear even if the behavior of the system is linear. This is typical of the design process where the structural properties which correlate loads and displacements are considered as design variables. Thus, the nonlinear law $\lambda=\lambda(p)$ can be drawn as in Fig.1.a, which shows that for each value of *p*, there is a corresponding value of λ. However, from Fig.1.b it is also clear that the response interval [λ_{min} λ_{max}] corresponding to [p_{min} p_{max}] cannot be simply obtained from $\lambda(p_{min})$ and $\lambda(p_{max})$. The problem of finding the interval response can be instead properly formulated as an optimization problem by assuming the objective function to be maximized as the size of the response interval itself. In particular, for the general case of *n* independent parameter *p*, collected in a vector $\mathbf{x}=[\,p_1 \;\; p_2 \;\; \dots \;\; p_n\,]^T$, and *m* assigned limit states, the following objective function is introduced:

$$F(\mathbf{x})=\sum_{i=1}^{m}\left(\lambda_{i,\max}-\lambda_{i,\min}\right) \tag{1}$$

A solution **x** of the optimization problem which take the side constraints $\mathbf{x}_{min} \le \mathbf{x} \le \mathbf{x}_{max}$ into account is developed by genetic algorithms, which are heuristic search techniques which belong to the class of stochastic algorithms, since they combine elements of deterministic and probabilistic search (Michalewicz 1992). The search strategy works on a *population* of *individuals* subjected to an evolutionary process where individuals compete between them to survive in proportion to their *fitness* with the *environment*. In this process, population undergoes continuous reproduction by means of some *genetic operators* which, because of competition, tend to preserve best individuals. From this evolutionary mechanism, two conflicting trends appear: exploiting of the best individuals and exploring the environment. Thus, the effectiveness of the genetic search depends on a balance between them, or between two principal properties of the system, *population diversity* and *selective pressure*. These aspects are in fact strongly related, since an increase in the selective pressure decreases the diversity of the population, and vice versa (Biondini 1999).

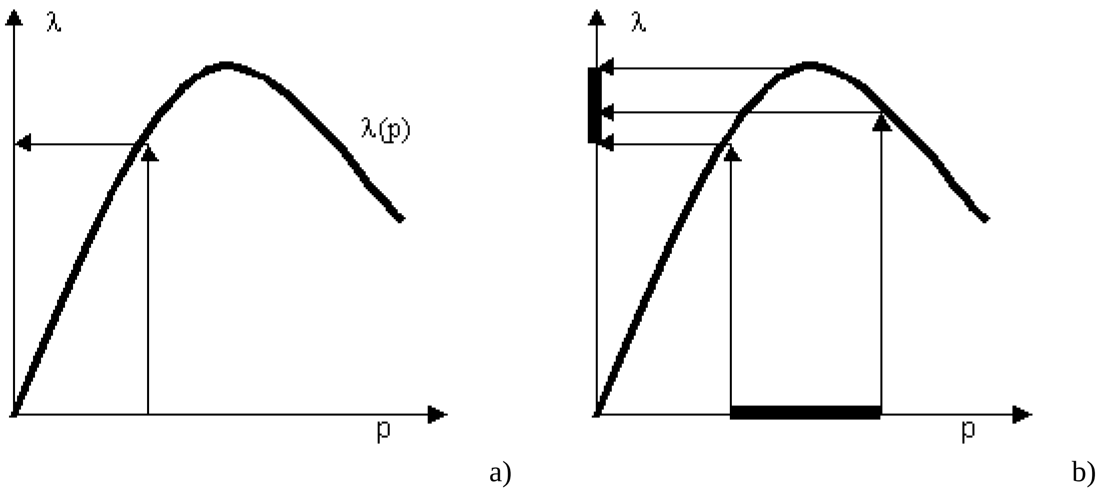


Figure 1. (a) Relationship between a structural parameter *p* and a limit state load multiplier λ. (b) Interval of the limit state multiplier λ corresponding to an interval of the parameter p.

3 NON-LINEAR ANALYSIS OF MASONRY STRUCTURES

In the following, a finite element suitable to model the structural behavior of masonry structures formed by rigid blocks of stone connected by layers of mortar, is described (Bolognini et al., 1994). The finite element, shown in Fig. 2, has total length l and is composed by one deformable layer of thickness l_i and by two rigid sub-elements of length $a=(l-l_i)/2$. The interfaces between sub-elements a and layers l_i are plane sections perpendicular to the axis of the elements.

3.1 *Kinematic field of the finite element*

The finite element is subjected to plane displacements and the kinematic field is completely defined by the displacements of the nodes 1 and 2 (Fig. 2), collected in the following vector:

$$\mathbf{q} = \begin{bmatrix} q_1 & q_2 & q_3 & q_4 & q_5 & q_6 \end{bmatrix}^T \tag{2}$$

Since the sub-elements are considered to be rigid, under the hypothesis of small displacements the displacements of the nodes 1′ and 2′ can be derived in the following matrix form:

$$\mathbf{q}' = \mathbf{C_g q} \qquad \Rightarrow \qquad \mathbf{q} = \mathbf{C_g}^{-1}\mathbf{q}' \tag{3}$$

where $\mathbf{C_g}$ is a geometrical matrix and $\mathbf{q}'$ is the vector of the nodal displacements of the layer:

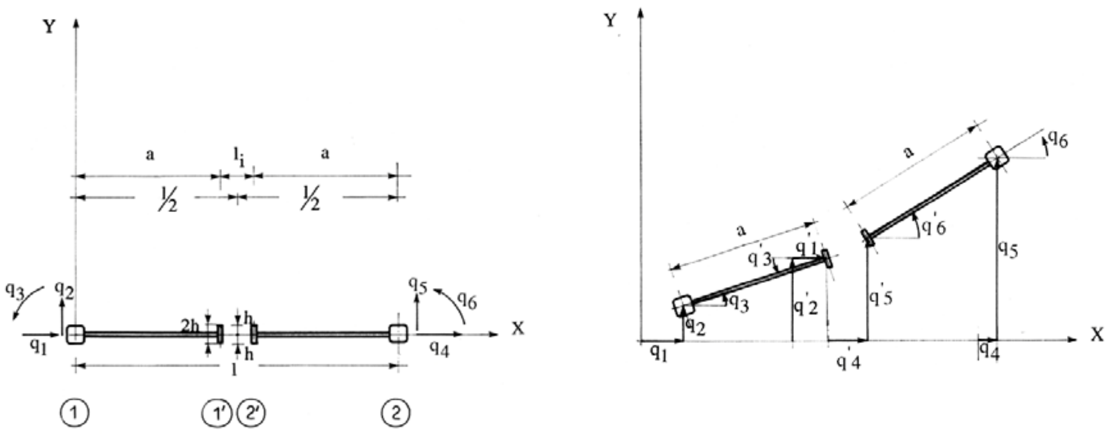
$$\mathbf{C_g} = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & & & \\ 0 & 1 & a & & \mathbf{0} & \\ 0 & 0 & 1 & & & \\ \hline & & & 1 & 0 & 0 \\ & \mathbf{0} & & 0 & 1 & -a \\ & & & 0 & 0 & 1 \end{array}\right] \qquad \mathbf{q}' = \begin{bmatrix} q'_1 & q'_2 & q'_3 & q'_4 & q'_5 & q'_6 \end{bmatrix}^T \tag{4}$$

Based on the following relative displacements:

$$\Delta n = q'_4 - q'_1 \qquad \Delta r = q'_6 - q'_3 \qquad \Delta t = q'_5 - q'_2 \tag{5}$$

the average strains of the layer can be derived:

$$\bar{\varepsilon} = \frac{\Delta n}{l_i} \qquad \bar{\chi} = \frac{\Delta r}{l_i} \qquad \bar{\gamma} = \frac{\Delta t}{l_i} \tag{6}$$



(a) (b)

Figure 2. Characteristics of the masonry finite element. (a) Local and (b) global reference systems.

By putting:

$$\Delta\mathbf{q} = \begin{bmatrix} \Delta n \\ \Delta r \\ \Delta t \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 & 1 & 0 \end{bmatrix} \mathbf{q}' = \mathbf{S}\mathbf{q}' \tag{7}$$

the strains at a distance *y* by the *x*-axis can be then described as follows:

$$\mathbf{e}(y) = \begin{bmatrix} \varepsilon(y) \\ \gamma(y) \end{bmatrix} = \begin{bmatrix} 1 & -y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{\varepsilon} \\ \bar{\chi} \\ \bar{\gamma} \end{bmatrix} = \mathbf{L} \begin{bmatrix} \bar{\varepsilon} \\ \bar{\chi} \\ \bar{\gamma} \end{bmatrix} = \frac{1}{l_i}\mathbf{L} \begin{bmatrix} \Delta n \\ \Delta r \\ \Delta t \end{bmatrix} = \frac{1}{l_i}\mathbf{L}\mathbf{S}\mathbf{q}' = \frac{1}{l_i}\mathbf{L}\mathbf{S}\mathbf{C}_\mathbf{g}\mathbf{q} = \frac{1}{l_i}\mathbf{K}_\mathbf{g}\mathbf{q} \tag{8}$$

where is the following geometrical matrix:

$$\mathbf{K}_\mathbf{g} = \mathbf{L}\mathbf{S}\mathbf{C}_\mathbf{g} = \begin{bmatrix} -1 & 0 & y & 1 & 0 & -y \\ 0 & -1 & -a & 0 & 1 & -a \end{bmatrix} \tag{9}$$

3.2 *Constitutive laws of the materials and stiffness matrix of the element*

A non-linear relation between the stains $\mathbf{e}(y) = [\varepsilon \;\; \gamma]^T$ and the correlative stress $\mathbf{s}(y) = [\sigma \;\; \tau]^T$ is assumed, being σ and τ the normal (along *x*-axis) and the shear (along *y*-axis) stress, respectively. The stress-strain relation can be written in secant form as follows:

$$\mathbf{s}(y) = \begin{bmatrix} E_s(\varepsilon) & \\ & G_s(\gamma) \end{bmatrix} \mathbf{e}(y) = \mathbf{D}\mathbf{e}(y) \tag{10}$$

where $E_s(\varepsilon)$ and $G_s(\gamma)$ are the longitudinal and the shear secant modulus of the deformable material, respectively. The stiffness matrix can be obtained by means of the Principle of Virtual Displacements. The internal virtual work is:

$$\delta L_i = \int_{-h}^{h} \delta\mathbf{e}^T \mathbf{s}\, dy = \delta\mathbf{q}^T \left(\frac{1}{l_i^2} \int_{-h}^{h} \mathbf{K}_\mathbf{g}{}^T \mathbf{D}\, \mathbf{K}_\mathbf{g}\, dy \right) \mathbf{q} = \delta\mathbf{q}^T \mathbf{K}\mathbf{q} \tag{11}$$

being $\mathbf{K}$ the secant stiffness matrix of the element, which can be written as follows:

$$\mathbf{K} = \frac{1}{l_i^2} \begin{bmatrix} I_1 & 0 & -I_2 & -I_1 & 0 & I_2 \\ & I_3 & aI_3 & 0 & -I_3 & aI_3 \\ & & I_4 + a^2 I_3 & I_2 & -aI_3 & -I_4 + a^2 I_3 \\ & & & I_1 & 0 & -I_2 \\ & & & & I_3 & -aI_3 \\ & & & & & I_4 + a^2 I_3 \end{bmatrix} \tag{12}$$

where:

$$I_1 = \int_{-h}^{h} E_s(y)dy \qquad I_2 = \int_{-h}^{h} yE_s(y)dy \qquad I_3 = \int_{-h}^{h} G_s(y)dy \qquad I_4 = \int_{-h}^{h} y^2 E_s(y)dy \tag{13}$$

The external virtual work done by the vector of the nodal forces **F** is:

$$\delta L_e = \delta\mathbf{q}^T \mathbf{F} \tag{14}$$

By equating the virtual works and by assembling the stiffness matrix **K** and the vector of the nodal forces **F** with reference to a global coordinate system, the equilibrium of the whole structure can be formally expressed as follows:

$$\mathbf{F} = \mathbf{K}\mathbf{q} \tag{15}$$

where **q** is the vector of the nodal displacements.

4 APPLICATION TO AN ARCH BRIDGE

The proposed procedure is applied to the fuzzy nonlinear structural analysis of the main arch of a masonry bridge having span length equal 27.00 m, radius equal 20.90 m, width equal 2.80 m, and thickness varying from 0.75 m on the top to 0.90 m at the lateral supports. The arch is discretized by using 28 equally spaced finite elements. The mortar is considered to be a no-tension material whose nonlinear behavior in compression is described by the parable-rectangle diagram usually used for concrete. Such diagram is defined by the strain values $\varepsilon_{c1} = -2‰$ and $\varepsilon_{cu} = -3.5‰$ of the horizontal branch and by the nominal strength of the mortar $f_{c,nom} = -20$ MPa. A brittle frictional sliding according to a Coulomb law with friction coefficient μ=0.2 is also assumed. The weight density of the stone was $\gamma = 22$ kN/m^3. Besides the self-weight of the structure, the structure is considered to be subjected to a moving live load $w = 25$ kN/m whose intensity λw is progressively increased until the limit state of collapse.

To show the capabilities of the procedure, the following quantities are considered uncertainty:

- the strength of the mortar (1 variable);
- the thickness of the joints (1 variable);
- the radius of the arch (1 variable);
- the shape of the profile, defined by a superposition of four sinusoids (4 variables);
- the distribution of the live load, which is assumed to be or not to be present on each couple of adjacent elements (14 variables).

These 21 fuzzy variables are assumed to have a triangular membership function, with interval base [0.50 – 1.50], and mean value equal 1.00. Four α-levels are considered, corresponding to the following intervals: [0.50 – 1.50], [0.625 – 1.375], [0.75 – 1.25], [0.875 – 1.125].

The minimum values of the load multiplier associated to the limit state of collapse for the considered α - levels are listed in Tab.1. With reference to the worst solution for the α-level 4, Fig.3 shows the nominal and the actual profiles of the structural model, the evolution of the deformed shape and of the internal forces diagrams with the increase of the load level, and the distribution of compressed material at collapse. A three hinges failure mechanism is clearly highlighted.

The effectiveness of the genetic algorithm in driving the simulation process towards the exploiting of the worst structural configuration can be appreciated from the inspection of Figg.4,5 and 6, which show for the first two and the last two generations of the search process the distribution of the perturbation of the arch profile, of both the strength of the mortar and the thickness of the joints, and of the location of the load, respectively. In particular, from these distributions can be noticed that the genetic search, starting from populations which tend to be randomly distributed, leads to found structural configurations characterized by loads applied on half arch, higher profile of the loaded zone and lower profile of the unloaded one, lower values of the strength of the mortar and higher values of the thickness of the joints. In other words, the proposed procedure is shown to be effective in finding the more dangerous configurations and the corresponding values of the collapse limit multipliers needful for a safety verification of the structure.

Table 1. Intervals of the α - levels and limit multipliers λ at collapse.

α-level	Lower limit	Upper limit	Limit multiplier
1	0.500	1.500	6.3
2	0.625	1.375	6.4
3	0.750	1.250	7.1
4	0.875	1.125	7.4

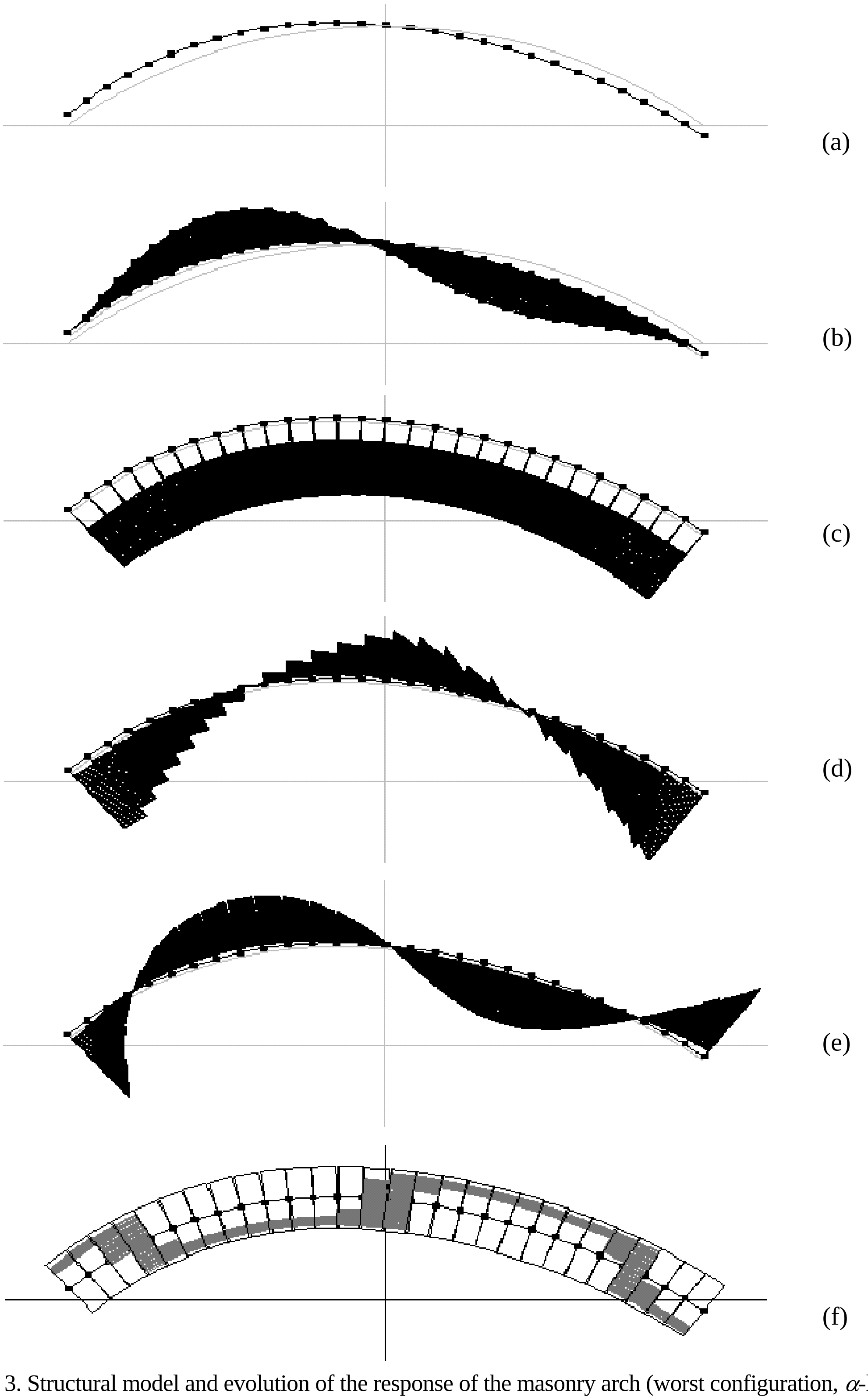
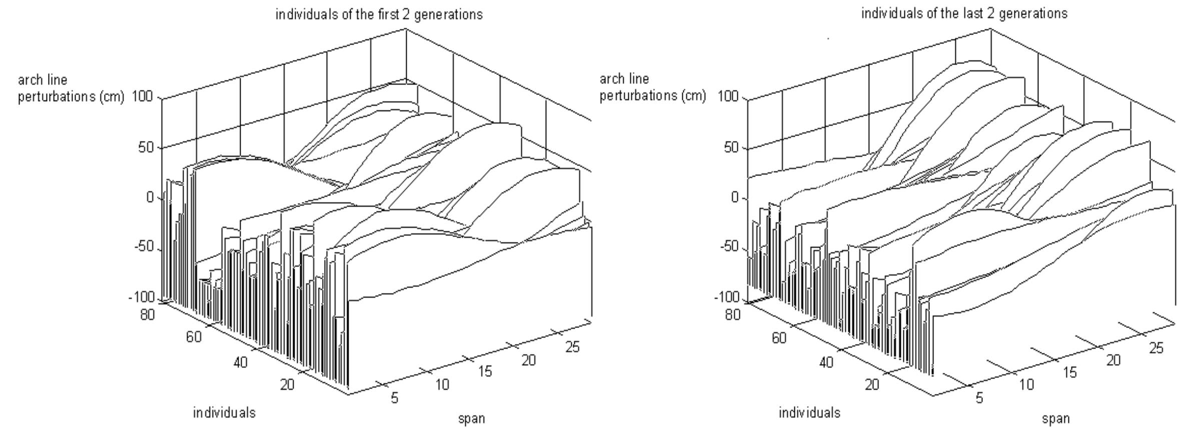
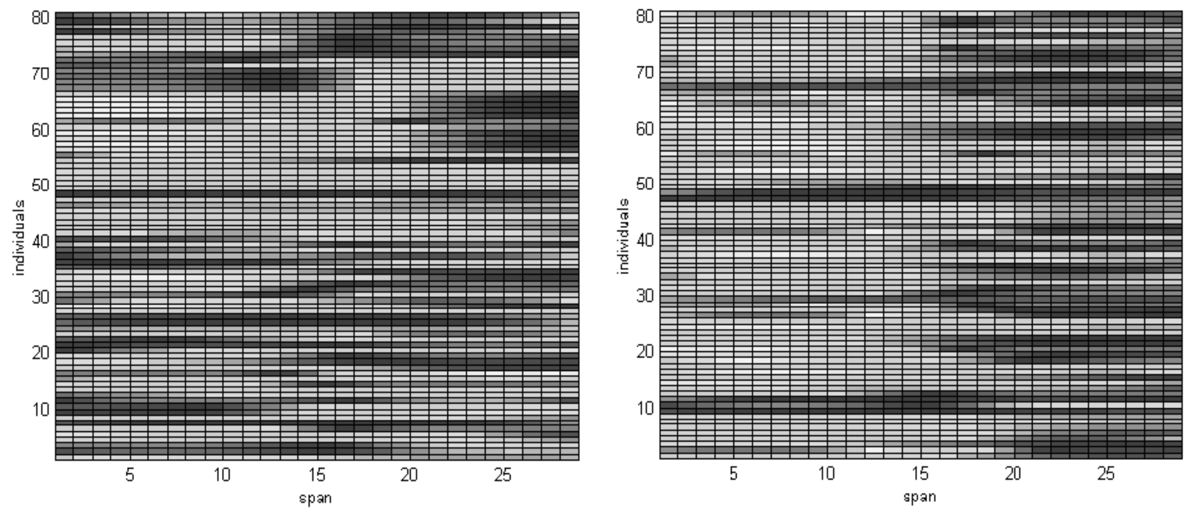


Figure 3. Structural model and evolution of the response of the masonry arch (worst configuration, α-level 4). (a) Nominal and actual profiles. Evolution of (b) deformed shape, (c) normal force, (d) shear force and (e) bending moment diagram. (f) Distribution of compressed material (shaded area) at collapse.



(a)



(b)

Figure 4. Distribution of the perturbation of the arch for the first two and the last two generations (α-level 4). (a) Three-dimensional diagram and (b) two-dimensional contour map of the perturbations along the arch.

(a) (b)

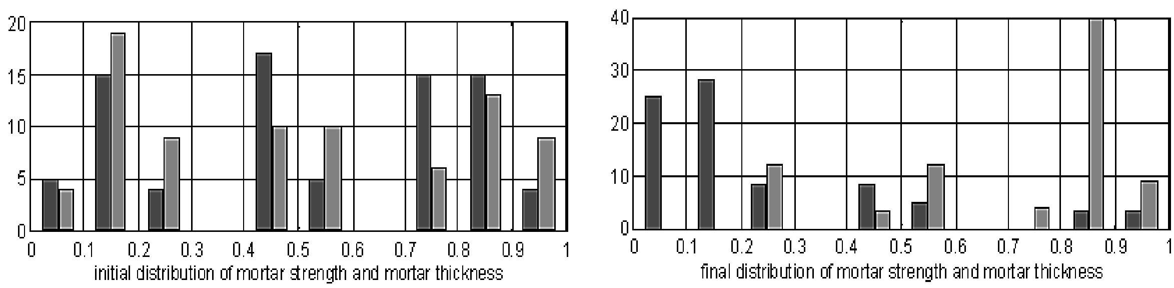
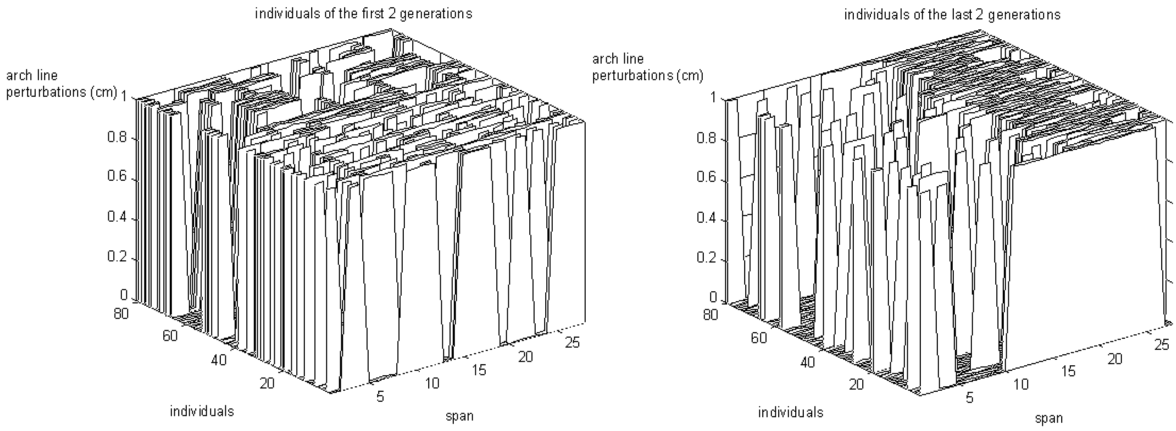
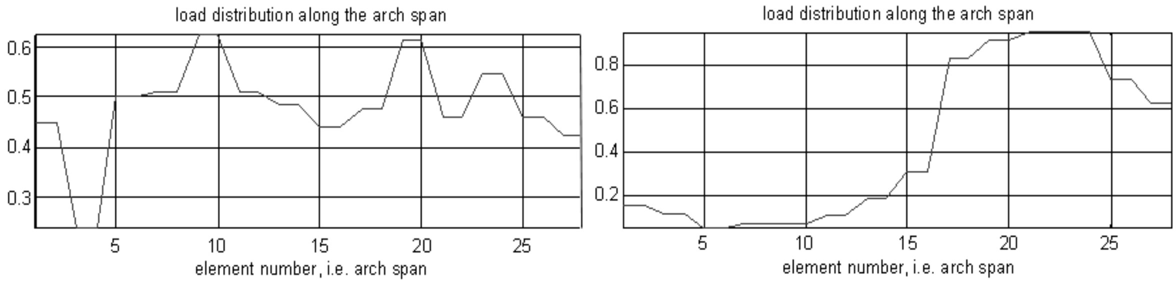


Figure 5. Distribution of the strength of the mortar (black histogram) and of the thickness of the joints (gray histogram) for (a) the first two and (b) the last two generations (α-level 4).



(a)



(b)

Figure 6. Distribution of the location of the live load for the first two and the last two generations (α-level 4). (a) Three-dimensional diagram and (b) average distribution of the load along the arch.

5 CONCLUSIONS

In this paper the effectiveness of the fuzzy non-linear analysis in a typical engineering problem having practical consequence has been investigated. The results show like the procedure is effective in finding the more dangerous configurations and the corresponding values of the collapse limit multipliers required for a safety verification of masonry bridge structures.

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